Soft Generalized Separation Axioms in Soft Generalized Topological Spaces

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Abstract: In this paper, the concepts of soft generalized μ-T_0, soft generalized μ-T_1, soft generalized Hausdorff, soft generalized regular, soft generalized normal and soft generalized completely regular spaces in Soft Generalized Topological Spaces are defined and studied. Further some of its properties and characterizations are established.

Keywords: Soft set, Soft Generalized Topological Space, soft generalized μ-T_0, soft generalized μ-T_1, soft generalized Hausdorff, soft generalized regular, soft generalized normal, soft generalized completely regular spaces.

1 Introduction

Several set theories can be considered as mathematical tools for dealing with uncertainties, namely, the theory of fuzzy sets [1], the theory of intuitionistic fuzzy sets [2], [3], the theory of vague sets [4], the theory of interval mathematics [5] and the theory of rough sets [6]. Molodtsov [7] in 1999, introduced the concept of soft set theory as a general mathematical tool to deal with uncertainties while modeling the problems with incomplete information. Many researchers improved the concept of soft sets. Maji et. al. [8] defined operations of soft sets. Pie and Miao [9] showed that soft sets are a class of special information systems. Cagman and Enginoglu [10] redefined the operations of the soft sets and constructed a uni-int decision making method by using these new operations and developed soft set theory. Aktas and Cagman [11] compared soft sets to fuzzy sets and rough sets. Babitha and Sunil [12] introduced the soft set relation and discussed related concepts such as equivalent soft set relation, partition and composition. Kharal and Ahmad [13] defined soft images and soft inverse images of soft sets. They also applied these notions to the problem of medical diagnosis in medical systems. Topological structure of soft sets also was studied by many researchers. Cagman [14] studied the concepts of soft topological spaces and some related concepts. Varol et al. [15] interpreted a classical topology as a soft set over the power set $\mathcal{P}$ (X) and characterized also some other categories related to crisp topology and fuzzy topology as subcategories of the category of soft sets.

General Topology was developed by many researchers. A Csaszar [16] introduced the theory of generalized topological spaces. Jyothis and Sunil [17], [18] introduced the concept of Soft Generalized Topological Space (SGTS) and studied the Soft μ-compactness in SGTSs. The generalized topology is different from topology by its axioms. According to Csaszar, a collection of subsets of X is a generalized topology on X if and only if it contains the empty set and arbitrary union of its members. But the soft generalized topology is based on soft sets theory and not sets. Some other studies on GTS's can be listed as [19], [20], [21].

This paper is organized as follows. In the second section, we give as a preliminaries, some well-known results in soft set theory and SGTS's. In section three, we introduce the concept of soft generalized separation axioms in SGTS's and discuss some of its properties and characterizations. We also investigate the behavior some soft generalized separation axioms under the soft continuous, soft open and soft closed mappings.

2. PRELIMINARIES

In this section, we recall the basic definitions and results of soft set theory and SGTS's which will be needed in the sequel. Throughout this paper U denotes initial universe, E denotes the set of all possible parameters, (U) is the power set of U and A is a nonempty subset of E.

Definition 2.1. [14] A soft set F_A on the universe U is defined by the set of ordered pairs $F_A = \{(e, f_A(e)) \,/\, e \in E, f_A(e) \in \mathcal{P}(U)\}$, where $f_A : E \to \mathcal{P}(U)$ such that $f_A(e) = \emptyset$ if $e \notin A$. Here f_A is called an approximate function of the soft set F_A. The value of $f_A(e)$ may be arbitrary. Some of them may be empty, some may have nonempty intersection. The set of all soft sets over U with E as the parameter set will be denoted by $S(U)_E$ or simply S(U).

Definition 2.2. [14] Let $F_A \in S(U)$. If $f_A(e) = \emptyset$ for all $e \in E$, then F_A is called an empty soft set, denoted by $F_\emptyset$. $f_A(e) = \emptyset$ means that there is no element in U related to the parameter e in E. Therefore we do not display such elements in the soft sets as it is meaningless to consider such parameters.

Definition 2.3. [14] Let $F_A \in S(U)$. If $f_A(e) = U$ for all $e \in A$, then F_A is called an A-universal soft set, denoted by $F_{\tilde{A}}$. If A = E, then the A-universal soft set is called an universal soft set, denoted by $F_{\tilde{E}}$.

Definition 2.4. [14] Let $F_A, F_B \in S(U)$. Then F_B is a soft subset of F_A (or F_A is a soft superset of F_B), denoted by $F_B \subseteq F_A$, if $f_B(e) \subseteq f_A(e)$, for all $e \in E$.

Definition 2.5. [14] Let $F_A, F_B \in S(U)$. Then F_B and F_A are soft equal, denoted by $F_B = F_A$, if $f_B(e) = f_A(e)$, for all $e \in E$.

Definition 2.6. [14] Let $F_A, F_B \in S(U)$. Then, the soft union of F_A and F_B, denoted by $F_A \cup F_B$, is defined by the approximate function $f_{(A\cup B)}(e) = f_A(e) \cup f_B(e)$.

Definition 2.7. [14] Let $F_A, F_B \in S(U)$. Then, the soft intersec-

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tion of F_A and F_B, denoted by $F_A \cap F_B$, is defined by the approximate function $f_{(A\cap B)}(e) = f_A(e) \cap f_B(e)$. F_A and F_B are said to be soft disjoint if $F_A \cap F_B = F_\emptyset$.

Definition 2.8. [14] Let F_A, $F_B \in S(U)$. Then, the soft difference of F_A and F_B, denoted by $F_A \backslash F_B$, is defined by the approximate function $f_{(A\backslash B)}(e) = f_A(e) \setminus f_B(e)$.

Definition 2.9. [14] Let $F_A \in S(U)$. Then, the soft complement of F_A, denoted by F_A^c, is defined by the approximate function $f_A^c(e) = (f_A(e))^c$, where $(f_A(e))^c$ is the complement of the set $f_A(e)$, that is, $(f_A(e))^c = U \setminus f_A(e)$ for all $e \in E$. Clearly $(F_A^c)^c = F_A$, $F_\emptyset^c = F_E$, and $F_E^c = F_\emptyset$.

Definition 2.10. [14] Let $F_A \in S(U)$. The soft power set of F_A, denoted by (F_A), is defined by $(F_A) = \{F_{A_i} / F_{A_i} \subseteq F_A, i \in J\}$

Theorem 2.11. [14] Let F_A, F_B, $F_C \in S(U)$. Then,

(1) $F_A \cup F_A^c = F_E$

(2) $F_A \cap F_A^c = F_\emptyset$.

(3) $F_A \subseteq F_B \Rightarrow F_B^c \subseteq F_A^c$.

(4) $F_A \cap F_B = F_\emptyset \Leftrightarrow F_A \subset F_B^c$

(5) $F_A \cup F_B = F_B \cup F_A$.

(6) $F_A \cap F_B = F_B \cap F_A$.

(7) $(F_A \cup F_B)^c = F_A^c \cap F_B^c$.

(8) $(F_A \cap F_B)^c = F_A^c \cup F_B^c$.

(9) $F_A \cup (F_B \cap F_C) = (F_A \cup F_B) \cap (F_A \cup F_C)$.

(10) $F_A \cap (F_B \cup F_C) = (F_A \cap F_B) \cup (F_A \cap F_C)$.

Definition 2.12. [13] Let $S(U)_E$ and $S(V)_K$ be the families of all soft sets over U and V respectively. Let $\varphi : U \to V$ and $\chi : E \to K$ be two mappings. The soft mapping $\varphi_\chi : S(U)_E \to S(V)_K$ is defined as:

(1) Let F_A be a soft set in $S(U)_E$. The image of F_A under the soft mapping φ_χ is the soft set over V, denoted by $\varphi_\chi(F_A)$ and is defined by

$$\varphi_\chi(f_A)(k) = \begin{cases} \bigcup_{e \in \chi^{-1}(k) \cap A} \varphi(f_A(e)), & \text{if } \chi^{-1}(k) \cap A \neq \emptyset; \\ \emptyset, & \text{otherwise} \end{cases}$$

for all $k \in K$.

(2) Let G_B be a soft set in $S(V)_K$. The inverse image of G_B under the soft mapping φ_χ is the soft set over U, denoted by $\varphi_\chi^{-1}(G_B)$ and is defined by

$$\varphi_\chi^{-1}(g_B)(e) = \begin{cases} \varphi^{-1}(g_B(\chi(e))), & \text{if } \chi(e) \in B; \\ \emptyset, & \text{otherwise} \end{cases}$$

for all $e \in E$.

The soft mapping φ_χ is called soft injective, if φ and χ are injective. The soft mapping φ_χ is called soft surjective, if φ and χ are surjective. The soft mapping φ_χ is called soft bijective iff φ_χ is soft injective and soft surjective.

Theorem 2.13. [13] Let $S(U)_E$ and $S(V)_K$ be the families of all soft sets over U and V respectively. Let F_A, F_B, $F_{A_i} \in S(U)_E$ and G_A, G_B, $G_{B_i} \in S(V)_K$. For a soft mapping $\varphi_\chi : S(U)_E \to S(V)_K$ the following statements are true:

(1) If $F_B \subseteq F_A$, then $\varphi_\chi(F_B) \subseteq \varphi_\chi(F_A)$.

(2) $\varphi_\chi(F_\emptyset) = F_\emptyset$.

(3) $\varphi_\chi(\bigcup_{i\in J} F_{Ai}) = \bigcup_{i\in J}(\varphi_\chi(F_{Ai}))$.

(4) $\varphi_\chi(\bigcap_{i\in J} F_{Ai}) \subseteq \bigcap_{i\in J} (\varphi_\chi(F_{Ai}))$, equality holds if φ_χ is soft injective.

(5) $F_A \subseteq \varphi_\chi^{-1}(\varphi_\chi(F_A))$, equality holds if φ_χ is soft injective.

(6) $\varphi_\chi(\varphi_\chi^{-1}(F_A)) \subseteq F_A$, equality holds if φ_χ is soft surjective.

(7) If $G_B \subseteq G_A$, then $\varphi_\chi^{-1}(G_B) \subseteq \varphi_\chi^{-1}(G_A)$.

(8) $\varphi_\chi^{-1}(F_\emptyset) = F_\emptyset$.

(9) $\varphi_\chi^{-1}(G_B^c) = (\varphi_\chi^{-1}(G_B))^c$

(10) $\varphi_\chi^{-1}(\bigcup_{i\in J} G_{Bi}) = \bigcup_{i\in J}(\varphi_\chi^{-1}(G_{Bi}))$.

(11) $\varphi_\chi^{-1}(\bigcap_{i\in J} G_{Bi}) = \bigcap_{i\in J}(\varphi_\chi^{-1}(G_{Bi}))$.

SOFT GENERALIZED TOPOLOGİCAL SPACES

Definition 2.14. [17] Let $F_A \in S(U)$. A Soft Generalized Topology (SGT) on F_A, denoted by μ or μ_{F_A} is a collection of soft subsets of F_A having the following properties: (i) $F_\emptyset \in \mu$ and (ii) The soft union of any number of soft sets in μ belong to μ. The pair (F_A, μ) is called a Soft Generalized Topological Space (SGTS). Observe that $F_A \in \mu$ must not hold.

Definition 2.15. [17] Let $F_A \in S(U)$ and μ be the collection of all possible soft subsets of F_A, then μ is a SGT on F_A, and is called the discrete SGT on F_A.

Definition 2.16. [17] A soft generalized topology μ on F_A is said to be strong if $F_A \in \mu$.

Definition 2.17. [17] Let (F_A, μ) be a SGTS. Then, every element of μ is called a soft μ–open set. Note that $F_\emptyset$ is a soft μ–open set.

Definition 2.18. [17] Let (F_A, μ) be a SGTS and $F_B \subseteq F_A$. Then the collection $\mu_{F_B} = \{F_D \cap F_B / F_D \in \mu\}$ is called a Subspace Soft Generalized Topology (SSGT) on F_B. The pair (F_B, μ_{F_B}) is called a Soft Generalized Topological Subspace (SGTSS) of F_A.

Definition 2.19. [17] Let (F_A, μ) be a SGTS and $F_B \subseteq F_A$. Then F_B is said to be a soft μ-closed set if its soft complement F_B^c is a soft μ-open set.

Theorem 2.20. [17] Let (F_A, μ) be a SGTS. Then the following conditions hold:

(1) The universal soft set F_E is soft μ–closed.

(2) Arbitrary soft intersections of the soft μ–closed sets are soft μ–closed.

Definition 2.21. [17] Let (F_A, μ) be a SGTS and $F_B \subseteq F_A$. Then the soft μ-closure of F_B, denoted by $c(F_B)$ or $\overline{F_B}$ is defined as the soft intersection of all soft μ-closed super sets of F_B. Note that $\overline{F_B}$ is the smallest soft μ-closed set that containing F_B.

Theorem 2.22. [17] Let (F_A, μ) be a SGTS and $F_B \subseteq F_A$. F_B is a soft μ-closed set iff $F_B = \overline{F_B}$.

Theorem 2.23. [17] Let (F_A, μ) be a SGTS and F_G, $F_H \subseteq F_A$. Then

(1) $F_G \subseteq \overline{F_G}$

(2) $\overline{(\overline{F_G})} = \overline{F_G}$

(3) $F_G \subseteq F_H \Rightarrow \overline{F_G} \subseteq \overline{F_H}$

(4) $\overline{F_G} \cap \overline{F_H} \supseteq \overline{F_G \cap F_H}$

(5) $\overline{F_G} \cup \overline{F_H} \subseteq \overline{F_G \cup F_H}$

(6) $\alpha \in \overline{F_H} \Rightarrow$ every soft μ-open set F_G containing α soft intersect F_H

Remark 2.24. Converse of theorem 2.23.(6) is not true in gen-

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eral as shown in the following example.

Example 2.25. Let U ={u_1, u_2, u_3}, E = {x_1, x_2, x_3}, A = {x_1, x_2} ⊆ E and F_A = {(x_1, {u_1, u_2}), (x_2, {u_2, u_3})} and μ = {$F_\emptyset$, F_A, F_P, F_Q, F_R}. Then (F_A, μ) is a SGTS where F_P = {(x_1, {u_2})}, F_Q = {(x_1, {u_2}), (x_2, {u_3})}, F_R = {(x_1, {u_1, u_2 }), (x_2, {u_2})}. The soft μ-closed sets are: $F_\emptyset^c$ = F_E, F_A^c = {(x_1, {u_3}), (x_2, {u_1}), (x_3, U)}, F_P^c = {(x_1, {u_1, u_3}), (x_2, U), (x_3, U)}, F_Q^c = {(x_1, {u_1, u_3}), (x_2, {u_1, u_2}), (x_3, U)}, F_R^c = {(x_1, {u_3}), (x_2, {u_1, u_3}), (x_3, U)}. Let F_B = {(x_1, {u_1}), (x_2, {u_2, u_3})}. Then $\overline{F_B}$ = F_P^c = {(x_1, {u_1, u_3}), (x_2, U), (x_3, U)}. Take α = (x_1, {u_1, u_2}) then, F_R is a soft μ-open set containing α. Now $F_R \cap F_B$ = {(x_1, {u_1}), (x_2, {u_2})} ≠ $F_\emptyset$. But α ∉ $\overline{F_B}$. i.e, We can find a soft μ-open set F_R containing α soft intersect F_B and α ∉ $\overline{F_B}$.

Definition 2.26. [17] Let (F_A, μ) be a SGTS and α ∈ F_A. If there is a soft μ-open set F_B such that α ∈ F_B, then F_B is called a soft μ-open neighborhood or soft μ-nbd of α. The set of all soft μ-nbds of α, denoted by ψ(α), is called the family of soft μ-nbds of α. i.e, ψ(α) = {F_B / F_B ∈ μ, α ∈ F_B}.

Definition 2.27. [17] Let (F_A, μ) and (F_B, η) be two SGTS's and φ_χ : (F_A, μ) → (F_B, η) be a soft function. Then

1. φ_χ is said to be soft (μ, η)-continuous (briefly, soft continuous), if for each soft η-open subset F_G of F_B, the inverse image $\varphi_\chi^{-1}(F_G)$ is a soft μ-open subset of F_A.
2. φ_χ is said to be soft (μ, η)-open, if for each soft μ-open subset F_G of F_A, the image $\varphi_\chi(F_G)$ is a soft η-open subset of F_B.
3. φ_χ is said to be soft (μ, η)-closed, if for each soft μ-closed subset F_G of F_A, the image $\varphi_\chi(F_G)$ is a soft η-closed subset of F_B.

Theorem 2.28. [17] Let (F_A, μ) and (F_B, η) be two SGTS's and φ_χ : (F_A, μ) → (F_B, η) be a soft function. Then φ_χ is soft continuous if and only if for every soft η-closed subset F_H of F_B, the soft set $\varphi_\chi^{-1}(F_H)$ is soft μ-closed in F_A.

3. SOFT GENERALİZED SEPARATION AXIOMS IN SGTSs

Definition 3.1. Let (F_A, μ) be a SGTS and α, β ∈ F_A such that α ≠ β. If there exists soft μ-open sets F_G and F_H such that α ∈ F_G and β ∉ F_G or β ∈ F_H and α ∉ F_H, then (F_A, μ) is called a soft generalized μ-T_0 space.

Theorem 3.2. Let (F_A, μ) be a SGTS and α, β ∈ F_A such that α ≠ β. If there exists soft μ-open sets F_G and F_H such that α ∈ F_G and β ∈ F_G^c or β ∈ F_H and α ∈ F_H^c, then (F_A, μ) is a soft generalized μ-T_0 space.

Proof: Let α, β ∈ F_A such that α ≠ β and F_G, F_H ∈ μ such that α ∈ F_G and β ∈ F_G^c or β ∈ F_H and α ∈ F_H^c. If α ∈ F_H^c then α ∉ $(F_H^c)^c$ = F_H. Similarly if β ∈ F_G^c then β ∉ $(F_G^c)^c$ = F_G. Hence ∃ F_G, F_H ∈ μ such that α ∈ F_G and β ∉ F_G or β ∈ F_H and α ∉ F_H. Hence (F_A, μ) is a soft generalized μ-T_0 space. ■

Example 3.3. A discrete SGTS (F_E, μ) is a soft generalized μ-T_0 space, since every {α} is a soft μ-open set.

Theorem 3.4. Let φ_χ : (F_A, μ) → (F_B, η) be a soft (μ, η) continuous soft bijective function. If (F_B, η) is a soft generalized η-T_0 space, then (F_A, μ) is also a soft generalized μ-T_0 space.

Proof: Let (F_B, η) be a soft generalized η-T_0 space. Suppose α, β ∈ F_A such that α ≠ β. Since φ_χ is soft injective, ∃ γ, δ ∈ F_B such that γ = φ_χ(α), δ = φ_χ(β) and γ ≠ δ. Since (F_B, η) is a soft generalized η-T_0 space, ∃ F_G, F_H ∈ η such that γ ∈ F_G and δ ∉ F_G or δ ∈ F_H and γ ∉ F_H. This implies that φ_χ(α) ∈ F_G and φ_χ(β) ∉ F_G or φ_χ(β) ∈ F_H and φ_χ(α) ∉ F_H ⇒ α ∈ $\varphi_\chi^{-1}(F_G)$ and β ∉ $\varphi_\chi^{-1}(F_G)$ or β ∈ $\varphi_\chi^{-1}(F_H)$ and α ∉ $\varphi_\chi^{-1}(F_H)$. Since φ_χ is a soft (μ, η) continuous function, $\varphi_\chi^{-1}(F_G)$ and $\varphi_\chi^{-1}(F_H)$ are soft μ-open sets. Hence (F_A, μ) is a soft generalized μ-T_0 space.■

Theorem 3.5. Let φ_χ : (F_A, μ) → (F_B, η) be a soft (μ, η) open soft bijective function. If (F_A, μ) is a soft generalized μ-T_0 space, then (F_B, η) is a soft generalized η-T_0 space.

Proof: Suppose that (F_A, μ) is a soft generalized μ-T_0 space. Let α, β ∈ F_B such that α ≠ β. Since φ_χ is a soft bijective function ∃ γ, δ ∈ F_A such that α = φ_χ(γ), β = φ_χ(δ) and γ ≠ δ. Since (F_A, μ) is a soft generalized μ-T_0 space, ∃ F_G, F_H ∈ μ such that γ ∈ F_G and δ ∉ F_G or δ ∈ F_H and γ ∉ F_H. This implies that φ_χ (γ) ∈ $\varphi_\chi(F_G)$ and φ_χ(δ) ∉ $\varphi_\chi(F_G)$ or φ_χ(δ) ∈ $\varphi_\chi(F_H)$ and φ_χ(γ) ∉ $\varphi_\chi(F_H)$ ⇒ α ∈ $\varphi_\chi(F_G)$ and β ∉ $\varphi_\chi(F_G)$ or β ∈ $\varphi_\chi(F_H)$ and α ∉ $\varphi_\chi(F_H)$. Since φ_χ is a soft (μ, η) open function, both $\varphi_\chi(F_G)$ and $\varphi_\chi(F_H)$ are soft η-open sets. Hence (F_B, η) is a soft generalized η-T_0 space.■

Definition 3.6. Let (F_A, μ) be a SGTS and α, β ∈ F_A such that α ≠ β. If there exists soft μ-open sets F_G and F_H such that α ∈ F_G and β ∉ F_G and β ∈ F_H and α ∉ F_H, then (F_A, μ) is called a soft generalized μ-T_1 space.

Theorem 3.7. Let ($F_{\tilde{E}}$, μ) be a SGTS. If for each α ∈ F_E, {α} is a soft μ-closed set, then (F_E, μ) is a soft generalized μ-T_1 space.

Proof: Let α and β be two points of $F_{\tilde{E}}$ such that α ≠ β. Given that {α} and {β} are soft μ-closed sets. Then $\{α\}^c$ and $\{β\}^c$ are soft μ-open sets. Clearly α ∈ $\{β\}^c$ and β ∉ $\{β\}^c$ and β ∈ $\{α\}^c$ and α ∉ $\{α\}^c$. Hence ($F_{\tilde{E}}$, μ) is a soft generalized μ-T_1 space.■

Theorem 3.8. Let (F_A, μ) be a SGTS and α, β ∈ F_A such that α ≠ β. If ∃ F_G, F_H ∈ μ such that α ∈ F_G and β ∈ F_G^c and β ∈ F_H and α ∈ F_H^c, then (F_A, μ) is a soft generalized μ-T_1 space.

Proof: The proof is similar to the proof of theorem 3.2.

Theorem 3.9. Every soft generalized μ-T_1 space is a soft generalized μ-T_0 space.

Proof: Let (F_A, μ) be a soft generalized μ-T_1 space and α, β ∈ F_A such that α ≠ β. So there exists soft μ-open sets F_G and F_H such that α ∈ F_G and β ∉ F_G and β ∈ F_H and α ∉ F_H. Obviously then we have α ∈ F_G and β ∉ F_G or β ∈ F_H and α ∉ F_H. Hence (F_A, μ) is a soft generalized μ-T_0 space.■

Theorem 3.10. Let (F_A, μ) be a soft generalized μ-T_1 space and α ∈ F_A. Then for each soft μ-open set F_G with α ∈ F_G, {α} ⊆ ∩F_G.

Proof: Since α ∈ F_G for each soft μ-open set F_G, α ∈ ∩F_G. Then it is obvious that {α}⊆ ∩F_G■

Theorem 3.11. Let (F_A, μ) be a SGTS and α, β ∈ F_A such that α ≠ β. If ∃ F_G, F_H ∈ μ such that α ∈ F_G and {β} ∩ F_G = $F_\emptyset$ and β ∈ F_H and {α} ∩ F_H = $F_\emptyset$, then (F_A, μ) is a soft generalized μ-T_1 space.

Proof: Similar to the theorem 3.8.■

Theorem 3.12. Let φ_χ : (F_A, μ) → (F_B, η) be a soft (μ, η) continuous soft bijective function. If (F_B, η) is a soft generalized η-T_1

space, then (F_A, μ) is also a soft generalized μ-T_1 space.

Proof: Proof is similar to that of soft generalized μ-T_0 space.■

Theorem 3.13. Let $\varphi_\chi : (F_A, \mu) \to (F_B, \eta)$ be a soft (μ, η)-open soft bijective function. If (F_A, μ) is a soft generalized μ-T_1 space, then (F_B, η) is also a soft generalized η-T_1 space.

Definition 3.14. Let (F_A, μ) be a SGTS. If for all $\alpha_1, \alpha_2 \in F_A$ with $\alpha_1 \neq \alpha_2$, there exists $F_G \in \psi(\alpha_1)$ and $F_H \in \psi(\alpha_2)$ such that $F_G \cap F_H = F_\emptyset$, then (F_A, μ) is called a soft generalized μ-T_2 space or soft generalized Hausdorff space (SGHS).

Theorem 3.15. If (F_A, μ) be a SGHS and F_B is a non-soft empty soft subset of F_A containing finite number of points, then F_B is soft μ-closed.

Proof: Suppose that (F_A, μ) is a SGHS. Let us take $F_B = \{\alpha\}$. Now we show that F_B is soft μ-closed. If β is a point of F_A different from α, then since (F_A, μ) is a SGHS, $\exists\ F_G, F_H \in \mu$ such that $\alpha \in F_G$, $\beta \in F_H$ and $F_G \cap F_H = F_\emptyset$. Now $F_G \cap F_H = F_\emptyset \Rightarrow F_G \cap \{\beta\} = F_\emptyset \Rightarrow \alpha$ cannot belong to the soft μ-closure of the soft set $\{\beta\}$. As a result, the soft μ-closure of the soft set $\{\alpha\}$ is $\{\alpha\}$ itself. Hence F_B is soft μ-closed.■

Theorem 3.16. A soft generalized subspace of a SGHS is a SGHS.

Proof: Let (F_A, μ) be a SGHS and (F_B, μ_{F_B}) is a SGTSS of F_A. Let $\alpha, \beta \in F_B$ such that $\alpha \neq \beta$. Then, since $F_B \subset F_A$, $\exists\ \alpha_1$ and β_1 in F_A such that $\alpha_1 \neq \beta_1$ and $\{\alpha\} \subseteq \{\alpha_1\}$ and $\{\beta\} \subseteq \{\beta_1\}$. Since (F_A, μ) is SGHS, $\exists\ F_G, F_H \in \mu$ such that $\alpha_1 \in F_G$, $\beta_1 \in F_H$ and $F_G \cap F_H = F_\emptyset$. Then clearly $\alpha \in F_G \cap F_B$ and $\beta \in F_H \cap F_B$ and $(F_G \cap F_B) \cap (F_H \cap F_B) = (F_G \cap F_H) \cap F_B = F_\emptyset$. i.e., there exist two soft disjoint soft μ_{F_B}-open sets $F_G \cap F_B$ and $F_H \cap F_B$ containing α and β respectively. Hence (F_B, μ_{F_B}) is a SGH sub space.■

Theorem 3.17. Every SGHS is a soft generalized μ-T_1 space.

Proof: Let (F_A, μ) be a SGHS and $\alpha, \beta \in F_A$ such that $\alpha \neq \beta$. Then $\exists\ F_G, F_H \in \mu$ such that $\alpha \in F_G$, $\beta \in F_H$ and $F_G \cap F_H = F_\emptyset$. Since $F_G \cap F_H = F_\emptyset$, $\alpha \notin F_H$, and $\beta \notin F_G$. Thus $F_G, F_H \in \mu$ such that $\alpha \in F_G$, $\beta \notin F_G$ and $\beta \in F_H$ and $\alpha \notin F_H$. Hence (F_A, μ) is a soft generalized μ-T_1 space.■

Theorem 3.18. Let (F_A, μ) be a SGHS and $\alpha \in F_A$. Then $\{\alpha\} = \cap F_P$ for each soft μ-open set F_P with $\alpha \in F_P$.

Proof: Assume that there exists a $\beta \in F_A$ such that $\alpha \neq \beta$ and $\beta \in \cap F_P$; $F_P \in \mu$, $\alpha \in F_P$. Since (F_A, μ) is a SGHS, $\exists\ F_G, F_H \in \mu$ such that $\alpha \in F_G$, $\beta \in F_H$ and $F_G \cap F_H = F_\emptyset$. Now $F_G \cap F_H = F_\emptyset \Rightarrow F_G \cap \{\beta\} = F_\emptyset$. This is a contradiction to the assumption that $\beta \in \cap F_P$. Hence $\{\alpha\} = \cap F_P$ for each soft μ-open set F_P with $\alpha \in F_P$.■

Theorem 3.19. Let $\varphi_\chi : (F_A, \mu) \to (F_B, \eta)$ be a soft bijective soft (μ, η) open function. If (F_A, μ) is a SGHS then (F_B, η) is a SGHS.

Proof: Let $\alpha, \beta \in F_B$ such that $\alpha \neq \beta$. Since φ_χ is a soft bijective function, $\exists\ \gamma, \delta \in F_A$ such that $\alpha = \varphi_\chi(\gamma)$, $\beta = \varphi_\chi(\delta)$ and $\gamma \neq \delta$. Since (F_A, μ) is a SGHS, $\exists\ F_G, F_H \in \mu$ such that $\gamma \in F_G$, $\delta \in F_H$ and $F_G \cap F_H = F_\emptyset$. This implies that $\varphi_\chi(\gamma) \in \varphi_\chi(F_G)$, $\varphi_\chi(\delta) \in \varphi_\chi(F_H) \Rightarrow \alpha \in \varphi_\chi(F_G)$ and $\beta \in \varphi_\chi(F_H)$. Since φ_χ is a soft (μ, η)-open function, $\varphi_\chi(F_G)$ and $\varphi_\chi(F_H)$ are soft η-open sets. Again since φ_χ is a soft bijective, $\varphi_\chi(F_G) \cap \varphi_\chi(F_H) = \varphi_\chi(F_G \cap F_H) = \varphi_\chi(F_\emptyset) = F_\emptyset$. Hence the proof.■

Theorem 3.20. Let $\varphi_\chi : (F_A, \mu) \to (F_B, \eta)$ be a soft bijective soft (μ, η) continuous function. If (F_B, η) is a SGHS, then (F_A, μ) is also a SGHS.

Proof: Let $\alpha, \beta \in F_A$ such that $\alpha \neq \beta$. Since φ_χ is a soft bijective function, $\exists\ \gamma, \delta \in F_B$ such that $\alpha = \varphi_\chi^{-1}(\gamma)$, $\beta = \varphi_\chi^{-1}(\delta)$ and $\gamma \neq \delta$. Since (F_B, η) is a SGHS, $\exists\ F_G, F_H \in \eta$ such that $\gamma \in F_G$, $\delta \in F_H$ and $F_G \cap F_H = F_\emptyset$. Then $\varphi_\chi^{-1}(F_G)$ and $\varphi_\chi^{-1}(F_H)$ are soft μ-open sets, because φ_χ is soft (μ, η) continuous. Also $F_G \cap F_H = F_\emptyset \Rightarrow \varphi_\chi^{-1}(F_G \cap F_H) = \varphi_\chi^{-1}(F_\emptyset) \Rightarrow \varphi_\chi^{-1}(F_G) \cap \varphi_\chi^{-1}(F_H) = F_\emptyset$. Now $\gamma \in F_G$ and $\delta \in F_H \Rightarrow \varphi_\chi^{-1}(\gamma) \in \varphi_\chi^{-1}(F_G)$ and $\varphi_\chi^{-1}(\delta) \in \varphi_\chi^{-1}(F_H) \Rightarrow \alpha \in \varphi_\chi^{-1}(F_G)$ and $\beta \in \varphi_\chi^{-1}(F_H)$. Hence (F_A, μ) is a SGHS.■

Definition 3.21. Let (F_A, μ) be a SGTS. If for every point $\alpha \in F_A$ and every soft μ-closed set F_M such that $\alpha \notin F_M$, there exists two soft μ-open sets F_G and F_H such that $\alpha \in F_G$, $F_M \subseteq F_H$ and $F_G \cap F_H = F_\emptyset$, then (F_A, μ) is called a soft generalized regular space (SGRS).

Theorem 3.22. Let (F_A, μ) be a SGTS and let F_K be a soft μ-closed set and $\alpha \in F_A$ such that $\alpha \notin F_K$. If (F_A, μ) is a SGRS, then there exists soft μ-open set F_G such that $\alpha \in F_G$ and $F_G \cap F_K = F_\emptyset$.

Proof: Let F_K be a soft μ-closed set and $\alpha \in F_A$ such that $\alpha \notin F_K$. Since (F_A, μ) is a SGRS, $\exists\ F_G, F_H \in \mu$ such that $\alpha \in F_G$, $F_K \subset F_H$ and $F_G \cap F_H = F_\emptyset$. Now $F_G \cap F_H = F_\emptyset \Rightarrow F_G \cap F_K = F_\emptyset$. Hence the proof.■

Theorem 3.23. Let (F_A, μ) be a SGRS and $\alpha \in F_A$. Then

(i) For a soft μ-closed set F_K, $\alpha \notin F_K$ iff $\{\alpha\} \cap F_K = F_\emptyset$
(ii) For a soft μ-open set F_H, $\{\alpha\} \cap F_H = F_\emptyset \Rightarrow \alpha \notin F_H$.

Proof: (i) Suppose that (F_A, μ) be a SGRS and $\alpha \in F_A$. Let F_K be a soft μ-closed set such that $\alpha \notin F_K$. Then by theorem 3.22. $\exists\ F_G \in \mu$ such that $\alpha \in F_G$ and $F_G \cap F_K = F_\emptyset$. Since $\{\alpha\} \subseteq F_G$, we have $\{\alpha\} \cap F_K = F_\emptyset$. The converse part is obvious.
(ii) Obvious.■

Theorem 3.24. Let (F_A, μ) be a SGRS and $\alpha \in F_A$. Then for each soft μ-closed set F_K such that $\{\alpha\} \nsubseteq F_K$, there exists soft μ-open sets F_G and F_H such that $\{\alpha\} \subseteq F_G$, $F_K \subseteq F_H$ and $F_G \cap F_K = F_\emptyset$.

Proof: Assume that (F_A, μ) is a SGRS and $\alpha \in F_A$. Let F_K be a soft μ-closed set such that $\{\alpha\} \nsubseteq F_K$. Then $\alpha \notin F_K$. Since (F_A, μ) is a SGRS, $\exists\ F_G, F_H \in \mu$ such that $\alpha \in F_G$, $F_K \subseteq F_H$ and $F_G \cap F_H = F_\emptyset$. Since $\alpha \in F_G$, $\{\alpha\} \subseteq F_G$. Hence F_G and F_H are soft μ-open sets such that $\{\alpha\} \subseteq F_G$, $F_K \subseteq F_H$ and $F_G \cap F_H = F_\emptyset$.■

Theorem 3.25. Let (F_A, μ) is a SGRS. Then for every $\alpha \in F_A$ and every soft μ-open set F_D with $\alpha \in F_D$, there exists a soft μ-open set F_H such that $\alpha \in F_H \subset \overline{F_H} \subset F_D$.

Proof: Suppose that (F_A, μ) is a SGRS. Let $\alpha \in F_A$, and F_D be any soft μ-open set such that $\alpha \in F_D$. Then F_D^c is a soft μ-closed set such that $\alpha \notin F_D^c$. Since (F_A, μ) is a SGRS, there exists soft μ-open sets F_G and F_H such that $\alpha \in F_H$, $F_D^c \subset F_G$ and $F_G \cap F_H = F_\emptyset$. Now $F_G \cap F_H = F_\emptyset \Rightarrow F_H \subseteq F_G^c$. Also $F_D^c \subseteq F_G \Rightarrow F_G^c \subseteq F_D$. This implies that $\alpha \in F_H \subseteq \overline{F_H} \subseteq \overline{(F_G^c)} \subseteq F_G^c \subseteq F_D$. ■

Theorem 3.26. Let (F_A, μ) and (F_B, η) be SGTS's and $\varphi_\chi : (F_A, \mu) \to (F_B, \eta)$ be a soft bijective, soft (μ, η) continuous and soft (μ, η) closed map. If (F_B, η) is a SGRS, then (F_A, μ) is also a SGRS.

Proof: Assume that (F_B, η) is a SGRS. Let $\alpha \in F_A$ and F_K be any soft μ-closed set in F_A such that $\alpha \notin F_K$. Since φ_χ is a soft bijective function, $\exists\ \delta \in F_B$ such that $\varphi_\chi(\alpha) = \delta \Rightarrow \alpha = \varphi_\chi^{-1}(\delta)$. Also $\varphi_\chi(F_K)$ is a soft η-closed set in F_B, since φ_χ is a closed map. Now $\alpha \notin F_K \Rightarrow \varphi_\chi(\alpha) \notin \varphi_\chi(F_K) \Rightarrow \delta \notin \varphi_\chi(F_K)$. Since (F_B, η) is a SGRS, $\exists$ $F_G, F_H \in \eta$ such that $\delta \in F_G$, $\varphi_\chi(F_K) \subseteq F_H$ and $F_G \cap F_H = F_\emptyset$. Then $\varphi_\chi^{-1}(F_G)$ and $\varphi_\chi^{-1}(F_H)$ are soft μ-open sets, since φ_χ is a soft (μ, η) continuous map. Now $\delta \in F_G \Rightarrow \varphi_\chi^{-1}(\delta) \in \varphi_\chi^{-1}(F_G) \Rightarrow \alpha \in \varphi_\chi^{-1}(F_G)$; $\varphi_\chi(F_K) \subseteq F_H \Rightarrow \varphi_\chi^{-1}(\varphi_\chi(F_K)) \subseteq \varphi_\chi^{-1}(F_H) \Rightarrow F_K \subseteq \varphi_\chi^{-1}(F_H)$ and $\varphi_\chi^{-1}(F_G) \cap \varphi_\chi^{-1}(F_H) = \varphi_\chi^{-1}(F_G \cap F_H) = \varphi_\chi^{-1}(F_\emptyset) = F_\emptyset$, since φ_χ is a soft bijective map. Hence (F_A, μ) is a SGRS.■

Theorem 3.27. Let (F_A, μ) be a SGTS. Then (F_A, μ) is a SGRS iff for each $\alpha \in F_A$ and a soft μ-closed set F_K such that $\alpha \notin F_K$, there exists a soft μ-open set F_G such that $\alpha \in F_G$ and $\overline{F_G} \cap F_K = F_\emptyset$.

Proof: Suppose that (F_A, μ) is a SGRS. Let $\alpha \in F_A$ and F_K be a soft μ-closed set such that $\alpha \notin F_K$. Then there exists soft μ-open sets F_G and F_H such that $\alpha \in F_G$, $F_K \subseteq F_H$ and $F_G \cap F_H = F_\emptyset$. Now $F_G \cap F_H = F_\emptyset \Rightarrow F_G \subseteq F_H^c$ and $F_K \subseteq F_H \Rightarrow F_H^c \subseteq F_K^c$. Thus $F_G \subseteq F_H^c \subseteq F_K^c$. This implies that $\overline{F_G} \subset \overline{(F_H^c)} = F_H^c \subseteq F_K^c$. Therefore $\overline{F_G} \cap F_K = F_\emptyset$.

Conversely, suppose $\alpha \in F_A$ and F_K be a soft μ-closed set such that $\alpha \notin F_K$. Then by hypothesis there exists a soft μ-open set F_G such that $\alpha \in F_G$ and $\overline{F_G} \cap F_K = F_\emptyset$. Now $\overline{F_G} \cap F_K = F_\emptyset \Rightarrow F_K \subseteq (\overline{F_G})^c$. Also $F_G \subseteq \overline{F_G} \Rightarrow (\overline{F_G})^c \subseteq F_G^c \Rightarrow F_G \subseteq ((\overline{F_G})^c)^c$. Therefore $F_G \cap (\overline{F_G})^c = F_\emptyset$. Thus $\exists$ $F_G, (\overline{F_G})^c \in \mu$ such that $\alpha \in F_G$, $F_K \subseteq (\overline{F_G})^c$, $F_G \cap (\overline{F_G})^c = F_\emptyset \Rightarrow (F_A, \mu)$ is a SGRS.■

Theorem 3.28. Let $\varphi_\chi : (F_A, \mu) \to (F_B, \eta)$ be a soft (μ, η) continuous, soft (μ, η) open, soft bijective function. If (F_A, μ) is a SGRS then (F_B, η) is also a SGRS.

Proof: Let $\alpha \in F_B$ and F_K a soft η-closed set such that $\alpha \notin F_K$. Since φ_χ is soft bijective, $\exists\ \delta \in F_A$ such that $\alpha = \varphi_\chi(\delta)$. Again since φ_χ is soft (μ, η) continuous, $\varphi_\chi^{-1}(F_K)$ is a soft μ-closed set such that $\delta \notin \varphi_\chi^{-1}(F_K)$. Since (F_A, μ) is a SGRS, $\exists$ $F_G, F_H \in \mu$ such that $\delta \in F_G$, $\varphi_\chi^{-1}(F_K) \subseteq F_H$ and $F_G \cap F_H = F_\emptyset$. Then $\varphi_\chi(\delta) \in \varphi_\chi(F_G)$, $\varphi_\chi(\varphi_\chi^{-1}(F_K)) \subseteq \varphi_\chi(F_H)$ and $\varphi_\chi(F_G \cap F_H) = \varphi_\chi(F_\emptyset) \Rightarrow \alpha \in \varphi_\chi(F_G)$, $F_K \subseteq \varphi_\chi(F_H)$ and $\varphi_\chi(F_G) \cap \varphi_\chi(F_H) = F_\emptyset$, since φ_χ is soft bijective. Moreover $\varphi_\chi(F_G)$ and $\varphi_\chi(F_H)$ are soft η-open sets, because φ_χ is (μ, η)-open. Hence (F_B, η) is a SGRS.■

Definition 3.29. Let (F_A, μ) be a SGTS. If for every pair of soft disjoint soft μ-closed sets F_M and F_N, there exists two soft μ-open sets F_G and F_H such that $F_M \subseteq F_G$, $F_N \subseteq F_H$ and $F_G \cap F_H = F_\emptyset$. Then (F_A, μ) is called soft generalized normal space (SGNS).

Theorem 3.30. A SGTS (F_A, μ) is a SGNS iff for any soft μ-closed set F_K and a soft μ-open set F_D containing F_K, there exists soft μ-open set F_G such that $F_K \subseteq F_G$ and $\overline{F_G} \subseteq F_D$.

Proof: Let (F_A, μ) be a SGNS and F_K be a soft μ-closed set and F_D be a soft μ-open set such that $F_K \subseteq F_D$. Then F_K and F_D^c are soft disjoint soft μ-closed sets. Since (F_A, μ) is a SGNS, $\exists$ $F_G, F_H \in \mu$ such that $F_K \subseteq F_G$, $F_D^c \subseteq F_H$ and $F_G \cap F_H = F_\emptyset$. Now $F_G \cap F_H = F_\emptyset \Rightarrow F_G \subseteq F_H^c \Rightarrow \overline{F_G} \subseteq \overline{F_H^c} = F_H^c$. Also $F_D^c \subseteq F_H \Rightarrow F_H^c \subseteq (F_D^c)^c = F_D$. Hence $F_K \subseteq F_G$ and $\overline{F_G} \subseteq F_D$.

Conversely, suppose that F_M and F_N are two soft μ-closed sets with soft empty soft intersection. Then, since $F_M \cap F_N = F_\emptyset$, $F_M \subseteq F_N^c$. i.e., F_M is a soft μ-closed set and F_N^c is a soft μ-open set containing F_M. So by hypothesis, there exists soft μ-open set F_G such that $F_M \subseteq F_G$ and $\overline{F_G} \subseteq F_N^c$. Now $\overline{F_G} \subset F_N^c \Rightarrow F_N \subset (\overline{F_G})^c$. Also $F_G \subseteq \overline{F_G} \Rightarrow (\overline{F_G})^c \subset F_G^c \Rightarrow (F_G^c)^c \subset ((\overline{F_G})^c)^c \Rightarrow F_G \subset ((\overline{F_G})^c)^c \Rightarrow F_G \cap (\overline{F_G})^c = F_\emptyset$. Thus F_G and $(\overline{F_G})^c$ are soft μ-closed sets such that $F_M \subseteq F_G$, $F_N \subseteq (\overline{F_G})^c$ and $F_G \cap (\overline{F_G})^c = F_\emptyset$. Hence (F_A, μ) is a SGNS.■

Theorem 3.31. Let $\varphi_\chi : (F_A, \mu) \to (F_B, \eta)$ be a soft bijective function which is both soft (μ, η) continuous and soft (μ, η) open. If (F_A, μ) is a SGNS then (F_B, η) is also a SGNS.

Proof: Let F_M and F_N be a pair of soft η-closed sets in (F_B, η) such that $F_M \cap F_N = F_\emptyset$. Since φ_χ is a soft (μ, η) continuous function, $\varphi_\chi^{-1}(F_M)$ and $\varphi_\chi^{-1}(F_N)$ are soft μ-closed sets in (F_A, μ). Also $\varphi_\chi^{-1}(F_M) \cap \varphi_\chi^{-1}(F_N) = \varphi_\chi^{-1}(F_M \cap F_N) = \varphi_\chi^{-1}(F_\emptyset) = F_\emptyset$. Again since (F_A, μ) is a SGNS, $\exists$ $F_G, F_H \in \mu$ such that $\varphi_\chi^{-1}(F_M) \subseteq F_G$, $\varphi_\chi^{-1}(F_N) \subseteq F_H$ and $F_G \cap F_H = F_\emptyset \Rightarrow F_M \subseteq \varphi_\chi(F_G)$ and $F_N \subseteq \varphi_\chi(F_H)$, since φ_χ is soft surjective. Since φ_χ is soft (μ, η) open, $\varphi_\chi(F_G)$ and $\varphi_\chi(F_H)$ are soft η-open sets. Also $\varphi_\chi(F_G) \cap \varphi_\chi(F_H) = \varphi_\chi(F_G \cap F_H) = \varphi_\chi(F_\emptyset) = F_\emptyset$, since φ_χ is soft bijective. Hence $\varphi_\chi(F_G)$ and $\varphi_\chi(F_H)$ are soft η-open sets such that $F_M \subseteq \varphi_\chi(F_G)$, $F_N \subseteq \varphi_\chi(F_H)$ and $\varphi_\chi(F_G) \cap \varphi_\chi(F_H) = F_\emptyset$. Hence (F_B, η) is a SGNS.■

Theorem 3.32. If (F_E, μ) is a SGNS, then for every pair of soft μ-open sets F_D and F_P whose soft union is F_E, then there exists soft μ-closed sets F_M and F_N such that $F_M \subset F_D$, $F_N \subset F_P$ and $F_M \cup F_N = F_E$.

Proof: Suppose that (F_E, μ) is a SGNS. Let F_D and F_P be a pair of soft μ-open sets such that $F_D \cup F_P = F_E$. Then F_D^c and F_P^c are soft μ-closed sets. Also $F_D^c \cap F_P^c = (F_D \cup F_P)^c = F_E^c = F_\emptyset$. Since (F_E, μ) is a SGNS, $\exists$ $F_G, F_H \in \mu$ such that $F_D^c \subseteq F_G$, $F_P^c \subseteq F_H$ and $F_G \cap F_H = F_\emptyset$. Take $F_M = F_G^c$ and $F_N = F_H^c$. Then F_M and F_N are soft μ-closed sets. Also $F_M \cup F_N = F_G^c \cup F_H^c = (F_G \cap F_H)^c = F_E$. Since $F_D^c \subseteq F_G$, $F_P^c \subseteq F_H \Rightarrow F_G^c \subseteq F_D$ and $F_H^c \subseteq F_P$. Thus there exists soft μ-closed sets F_M and F_N such that $F_M \subseteq F_D$, $F_N \subseteq F_P$ and $F_M \cup F_N = F_E$.■

Definition 3.33. Let (F_A, μ) be a SGTS. If for any point $\alpha \in F_A$ and a soft μ-closed set F_B such that $\alpha \notin F_B$, there exists soft continuous function $\varphi_\chi : F_A \to F_{[0,1]}$ such that $\varphi_\chi(\alpha) = (0, F_{[0,1]}(0))$ and $\varphi_\chi(F_B) = (1, F_{[0,1]}(1))$ where $F_{[0,1]}$ is a SGHS. Then (F_A, μ) is called a soft generalized completely regular space (SGCRS).

Theorem 3.34. Every SGCRS is a SGRS.

Proof: Let (F_A, μ) be a SGCRS. Let $\alpha \in F_A$ and F_K is a soft μ-closed set such that $\alpha \notin F_K$. Since (F_A, μ) is a SGCRS, there exists a soft continuous function $\varphi_\chi : F_A \to F_{[0,1]}$ where $F_{[0,1]}$ is a SGHS and $\varphi_\chi(\alpha) = (0, F_{[0,1]}(0))$ and $\varphi_\chi(F_K) = (1, F_{[0,1]}(1))$. Let $\beta = (0, F_{[0,1]}(0))$ and $\gamma = (1, F_{[0,1]}(1))$. Since $F_{[0,1]}$ is a SGHS, there exists two soft sets F_G and F_H such that $\beta \in F_G$, $\gamma \in F_H$ and $F_G \cap F_H = F_\emptyset$. Let $F_P = \varphi_\chi^{-1}(F_G)$ and $F_Q = \varphi_\chi^{-1}(F_H)$. Since φ_χ is a soft continuous function, F_P and F_Q are soft μ-open sets. Then clearly $\alpha \in F_P$, $F_K \subset F_Q$ and $F_P \cap F_Q = F_\emptyset$. Hence (F_A, μ) is a SGRS.■

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